(x_1,y_1) (x_2,y_2) $\frac{y_2-y_1}{x_2-x_1}$

Differential Calculus

Suppose f is a real function and c is a point in its domain.
The derivative of f at c is defined by

$\frac{y_2-y_1}{x_2-x_1}$

$$\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

$f(c)$ $f(c+h)$ c $c+h$

$Lt_{h\to 0}\frac{f(c+h)-f(c)}{c+h-c}$

provided this limit exists. Derivative of f at c is denoted by
$f'(c)\ or\ \frac{}{dx}(f(x))\ at\ x=c$.

$f'(c)$, $\frac{d}{dx}(f(x))$ at $x=c$

The function defined by

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$y=f(x)$
$y'=f'(x)$
$\frac{dy}{dx}$ $\frac{df}{dx}$

wherever the limit exists is defined to be the derivative of f. The derivative of f is denoted by $f'(x)\ or\ \frac{}{dx}(f(x))$ or if $y=f(x)$ then y' or $\frac{}{dx}$.

The following rules were established as a part of algebra of derivatives:

(1) $(u\pm v)'=u'\pm v'$

(2) $(uv)'=u'v+uv'$ (Leibnitz or product rule)

(3) $\left(\frac{u}{v}\right)'=\frac{vu'-v'u}{v^2}$ (Quotient rule).

$f(x)=x^2$

$Lt_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=Lt_{h\to 0}\frac{(x+h)^2-x^2}{h}$

$Lt_{h\to 0}\frac{\cancel{x^2}+h^2+2xh-\cancel{x^2}}{h}$

$Lt_{h\to 0}\ h+2x$

$=0+2x$

$\frac{d}{dx}(x^2)=2x$ | $\frac{d}{dx}(x^3)=3x^2$

The following table gives a list of derivatives of certain standard functions:

$f(x)$	x^n	$\sin x$	$\cos x$	$\tan x$
$f'(x)$	nx^{n-1}	$\cos x$	$-\sin x$	$\sec^2 x$

nx^{n-1}

$\cot x$ → $-\operatorname{cosec}^2 x$

$\operatorname{cosec} x$ → $-\operatorname{cosec} x.\cot x$

Derivatives of composite functions

$\frac{d}{dx}f(g(x))$

$f'(g(x)).g'(x)$

$f\circ g(x)=f(g(x))$
$g\circ f(x)=g f(x)$

Chain Rule: If $f=v(u(x))$ then $\frac{df}{dx}=\frac{dv}{du(x)}*\frac{du(x)}{dx}$

$g \circ f(x) = g f(x)$

Chain Rule: If $f = v(u(x))$ then $\frac{df}{dx} = \frac{dv}{du(x)} * \frac{du(x)}{dx}$

or if we write t=u(x) then $\frac{df}{dx} = \frac{dv}{dt} * \frac{dt}{dx}$

or

If $f = w\left(u\left(v(x)\right)\right)$ then $\frac{df}{dx} = \frac{dw}{ds} * \frac{ds}{dt} * \frac{dt}{dx}$ where $t = v(x)$ and $s = u(t)$.

Ex. Find the derivative of $(2x+1)^3$

$f(x) = 2x+1 \quad g(x) = x^3$

$y = (2x+1)^3$

$y' = 3(2x+1)^2 \cdot 2$

$= 6(2x+1)^2$

Ex. Find the derivative of $\sin(x^3)$

$\cos(x^3) \cdot 3x^2$

$\boxed{3x^2 \cos x^3}$

$= \left(\sin(\log(x))\right)^{3/4}$

$\frac{3}{4}\left(\sin(\log x)\right)^{-1/4} \cdot \cos(\log x) \cdot \frac{1}{x}$

$\frac{3}{4} - 1$

$\cos($

$\cos(\tan\sqrt{x}) \sec^2\sqrt{x} \cdot \frac{1}{2\sqrt{x}}$

$\cos(\tan\sqrt{x}) \left(\frac{1}{2\sqrt{x}}\right) \sec^2\sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2+\sqrt{a^2+x^2}}} \cdot \left(0 + \frac{1}{\cancel{2}\sqrt{a^2+x^2}} \cdot \cancel{2}x\right)$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2+\sqrt{a^2+x^2}}}\cdot\left(0 + \frac{\cancel{2}x}{\cancel{2}\sqrt{a^2+x^2}}\right)$$

$$= \frac{1}{2y}\cdot\frac{x}{\sqrt{a^2+x^2}}$$

$$\frac{d}{dx}\big(f(g(x))\big)$$

$$= f'(g(x))\cdot g'(x)$$

If $f''(x) = -f(x)$, where f(x) is double differentiable function and $g(x) = f'(x)$. If $F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$ and F(5) = 5, then F(10) is

A 0 B 5 C 10 D 25

$F'(x) = 2\cdot f\left(\frac{x}{2}\right)\cdot f'\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$2\cdot f\left(\frac{x}{2}\right) f'\left(\frac{x}{2}\right)\cdot\frac{1}{2}$

$F(x) = \left(f\left(\frac{x}{2}\right)\right)^2 + \left(g\left(\frac{x}{2}\right)\right)^2$

$f''(x) = -f(x)$

$g(x) = f'(x)$

g'

$F'(x) = \cancel{2}\cdot f\left(\frac{x}{2}\right) f'\left(\frac{x}{2}\right)\cdot\frac{1}{\cancel{2}} + \cancel{2}\, g\left(\frac{x}{2}\right)\cdot g'\left(\frac{x}{2}\right)\cdot\frac{1}{\cancel{2}}$

$-f\left(\frac{x}{2}\right) f'\left(\frac{x}{2}\right)$ $\quad f'\left(\frac{x}{2}\right)$

a. $3x\cos(x^3)$
b. $3x^2\cos(x^3)$
c. $4x^2\cos(x^3)$
d. $3x^2\sin(x^3)$

Ex. Find the derivative of $\sin^{3/4}(\log(x))$

Ex. Find the derivative of $\sin(\tan\sqrt{x})$

Ex. Find the derivative of $y = \sqrt{a^2+\sqrt{a^2+x^2}}$

a. $\frac{1}{y}\frac{x}{}$

b. $\frac{1}{3y} * \frac{x}{\sqrt{a^2+x^2}}$

c. $\frac{1}{y}\frac{x}{\sqrt{a^2+x^2}}$

d. None of these

Derivatives of implicit functions

When a relationship between x and y is expressed in a way that it is easy to solve for y and write y = f (x), we say that y is given as an explicit function of x.

We ~~say that the~~ relationship of the type

$$x + \sin xy + y = 0$$

is implicit because its difficult to write y as a function of x.

1. $2x + 3y = \sin x$ 2. $2x + 3y = \sin y$ 3. $ax + by^2 = \cos y$

4. $xy + y^2 = \tan x + y$ 5. $x^2 + xy + y^2 = 100$ 6. $x^3 + x^2y + xy^2 + y^3 = 81$

Derivatives of inverse trigonometric functions

Ex. Find the derivatives of $y = \sin^{-1} x$

Ex. Find the derivatives of $y = \tan^{-1} x$

$f(x)$	$\cos^{-1}x$	$\cot^{-1}x$	$\sec^{-1}x$	$\operatorname{cosec}^{-1}x$
$f'(x)$	$\frac{-1}{\sqrt{1-x^2}}$	$\frac{-1}{1+x^2}$	$\frac{1}{x\sqrt{x^2-1}}$	$\frac{-1}{x\sqrt{x^2-1}}$

10. $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right), \quad -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

11. $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

12. $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right), 0 < x < 1$

13. $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), -1 < x < 1$

Q2. If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then $(1 - x^2)\frac{dy}{dx}$ is equal to

A. x + y

B. 1 + xy

C. 1 - xy

D. xy -2

Q6. If $\sin^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \log a$, then $\frac{dy}{dx}$ is equal to

A. $\frac{x}{y}$

B. $\frac{y}{x^2}$

C. $\frac{x^2-y^2}{x^2+y^2}$

D. $\frac{y}{x}$

Q9. If $f(x) = 2\sin^{-1}\sqrt{1-x} + \sin^{-1}\left(2\sqrt{x(1-x)}\right)$, where $x \in \left(0, \frac{1}{2}\right)$, then $f'(x)$ is

A $\frac{2}{\sqrt{x(1-x)}}$

B zero

C $-\frac{2}{\sqrt{x(1-x)}}$

D π

Q2. If f (1) = 1, f' (1)= 3, then the derivative of f (f (f (x))) + (f (x)^2) at x = 1 is:

$f(g(x))$ -
$f'(g(x)).g'(x)$

A 33

B 15

C 9

D 12

$f(f(f(x))) + f(x^2)$

$f'(f(f(x)))\, f'(f(x)) . f'(x) + f'(x^2). 2x$

$f'(f(f(1)))\, f'(f(1))\, f'(1) + f'(1).2.$

$3.3.3 + 2.3$

$27 + 6 = 33$

$f'(f(1))$

$f'(1) = 3$

Q7. If y = sec(tan^{-1}x), then $\frac{dy}{dx}$ at x = 1 is equal to

A $\frac{1}{2}$

B 1

C $\sqrt{2}$

D $\boxed{\frac{1}{\sqrt{2}}}$

$y = \sec(\tan^{-1}x)$

$\frac{dy}{dx} = \sec(\tan^{-1}x)\tan(\tan^{-1}x) . \frac{1}{1+x^2}$

$\frac{dy}{dx} = \sec(\tan^{-1}x) . x . \frac{1}{1+x^2}$

$\left(\frac{dy}{dx}\right)_{x=1} = \sec(\tan^{-1}(1)) . \frac{1}{1+1}$

$= \sec\left(\frac{\pi}{4}\right) . \frac{1}{1+1}$

$= \frac{1}{\cos(\frac{\pi}{4})} . \frac{1}{2} = \frac{1}{\frac{1}{\sqrt{2}}} . \frac{1}{2} = \frac{\sqrt{2}}{2}$

$= \frac{1}{\sqrt{2}}$

$\tan^{-1}(1) = \frac{\pi}{4}$

$\frac{d}{dx}\log(x) = \frac{1}{x}$

$\frac{d}{dx}\left(e^{f(x)}\right)$

$e^{f(x)} . f'(x)$

Derivative of exponential and Logarithmic Functions

(i) e^{-x} (ii) $\sin(\log x), x > 0$ (iii) $\cos^{-1}(e^x)$ (iv) $e^{\cos x}$

$y = e^{-x}$

$\frac{dy}{dx} = e^{-x}.(-1)$

$\boxed{-e^{-x}}$

$\cos(\log x) . \frac{1}{x}$

$\frac{-1}{\sqrt{1-(e^x)^2}} . e^x = \frac{-e^x}{\sqrt{1-e^{2x}}}$

$\to e^{\cos x}.(-\sin x)$

$= -\sin x . e^{\cos x}$

1. $\frac{e^x}{\sin x}$ 2. $e^{\sin^{-1} x}$ 3. e^{x^3}

4. $\sin(\tan^{-1} e^{-x})$ 5. $\log(\cos e^x)$ 6. $\left(e^x + e^{x^2} + ... + e^{x^5}\right)' = e^x + e^{x^2}.2x + e^{x^3}.3x^2$

1. $\dfrac{e^x}{\sin x}$ 2. $e^{\sin^{-1} x}$ 3. e^{x^3}

4. $\sin(\tan^{-1} e^{-x})$ 5. $\log(\cos e^x)$ 6. $\left(e^x + e^{x^2} + \ldots + e^{x^5}\right)' = e^x + e^{x^2}\cdot 2x + e^{x^3}\cdot 3x^2 + e^{x^4}\cdot 4x^3$

7. $\sqrt{e^{\sqrt{x}}},\ x > 0$ 8. $\log(\log x), x > 1$ 9. $\dfrac{\cos x}{\log x},\ x > 0$

10. $\cos(\log x + e^x), x > 0$

$\to \cos(\tan^{-1}(e^{-x}))\cdot \dfrac{1}{1+e^{-2x}}\cdot(-e^{-x})$

$y = e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5} + \ldots e^{x^n}$ $\quad e^{x^2} \neq e^{2x}$

$1 + r + r^2 + r^3 + \ldots$ $\quad e^{x^2} = (e^x)^2 = e^x \cdot e^x = e^{2x}$ $\quad \dfrac{e^{x^2}}{e^x} = e^{x^2 - x}$

$y =$

Logarithmic Differentiation

$$y = f(x) = [u(x)]^{v(x)}$$

1. $\log(a \cdot b) = \log a + \log b$
2. $\log\left(\frac{a}{b}\right) = \log a - \log b$
3. $\log(a^b) = b \log a$
4. $\log(e^{f(x)}) = f(x)$
5. $e^{\log(f(x))} = f(x)$

$$\log y = v(x) \log[u(x)]$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = v(x)\cdot\frac{1}{u(x)}\cdot u'(x) + v'(x)\cdot\log[u(x)]$$

$$\frac{dy}{dx} = y\left[\frac{v(x)}{u(x)}\cdot u'(x) + v'(x)\cdot\log[u(x)]\right]$$

Ex.

Differentiate $\sqrt{\dfrac{(x-3)(x^2+4)}{3x^2+4x+5}}$ w.r.t. x.

$$y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}} = \left(\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right)^{1/2}$$

$$\log y = \frac{1}{2}\left(\log(x-3) + \log(x^2+4) - \log(3x^2+4x+5)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-3} + \frac{1}{x^2+4}\cdot 2x - \frac{1}{3x^2+4x+5}\cdot(6x+4)\right)$$

$$\frac{dy}{dx} = \frac{y}{2}\left(\frac{1}{x-3} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right)$$

Ex.

Differentiate $\boxed{a^x}$ w.r.t. x, where a is a positive constant.

$y = a^x$ $\quad 2^x$

$9.x$ $\quad \log y = x \log a$ $\quad 2^x \cdot \log 2$

2^x

$2^x \cdot \log 2$

$\log y = x \log a$

$\frac{1}{y} \cdot \frac{dy}{dx} = \log a$

$\frac{dy}{dx} = y \cdot \log a = \boxed{a^x \cdot \log a}$

$2x$

$2 \cdot 1 + x \cdot 0$

2

Ex.

Differentiate $x^{\sin x}$, $x > 0$ w.r.t. x.

$y = x^{\sin x}$

$\log y = \sin x \cdot \log x$

$\frac{1}{y} \cdot \frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \log x \cdot \cos x$

$\frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cdot \cos x\right)$

$\boxed{Lt_{x \to 0} \cancel{\frac{\sin x}{x}} = 1}$

$\log(a+b) \neq \log a + \log b$

$(u \pm v)' = u' \pm v'$

Ex.

Find $\frac{dy}{dx}$, if $(y^x + x^y + x^x)' = (a^b)'$

$= 0$

$x \log y$

$(y^x)' + (x^y)' + (x^x)'$

$D_1' \quad D_2' \quad D_3'$

$D_1 = y^x$

$\log D_1 = x \log y \quad \Big| \quad \frac{1}{D_1} \cdot \frac{dD_1}{dx} = x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \quad \Big| \quad D_1' = D_1\left(\frac{x}{y}\boxed{\frac{dy}{dx}} + \log y\right)$

Q10. If $x^m y^n = (x+y)^{m+n}$, $\frac{dy}{dx}$ then is

A $\frac{x}{y}$

B $\frac{y}{x}$

C $\frac{x+y}{xy}$

D xy

$m \log x + n \log y = (m+n) \log (x+y)$

$\frac{m}{x} + \frac{n}{y} \cdot \frac{dy}{dx} = (m+n) \frac{1}{x+y} \cdot \left(1 + \frac{dy}{dx}\right)$

$\left(\frac{n}{y} - \frac{m+n}{x+y}\right) \frac{dy}{dx} = \frac{m+n}{x+y} - \frac{m}{x}$

$\left(\frac{nx + \cancel{ny} - my - \cancel{ny}}{y(x+y)}\right) \frac{dy}{dx} = \frac{\cancel{xm} + xn - \cancel{xm} - my}{x(x+y)}$

$\left(\frac{nx - my}{y}\right) y' = \frac{nx - my}{x}$

$y' = \frac{y}{x}$

1. $\cos x \cdot \cos 2x \cdot \cos 3x$

2. $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

3. $(\log x)^{\cos x}$

4. $x^x - 2^{\sin x}$

u A v B

5. $(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

6. $\left(x+\dfrac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Just a Min

7. $(\log x)^x + x^{\log x}$

8. $(\sin x)^x + \sin^{-1}\sqrt{x}$

9. $x^{\sin x} + (\sin x)^{\cos x}$

10. $x^{x\cos x} + \dfrac{x^2+1}{x^2-1}$

11. $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Q5. The function f(x) = $e^x + x$, being differentiable and one to one, has a differentiable inverse $f^{-1}(x)$. The value of $\frac{d}{dx}(f^{-1})$ at the point f(log 2) is

A $\dfrac{1}{\ln 2}$

B $\dfrac{1}{3}$

C $\dfrac{1}{4}$

D None of these

$f'(x) = e^x + 1$

$f(x) = e^x + x$

$\frac{d}{dx}(f^{-1}(x))$ $\boxed{x = f(\log 2)}$

$f(f^{-1}(x)) = x$

Diff w.r.t x

$f'(f^{-1}(x))(f^{-1}(x))' = 1$

$f^{-1}(x) = g$

$$f(g(x)) = x$$
$$\Rightarrow f'(g(x))g'(x) = 1$$
$$\Rightarrow (e^{g(f(\log 2))} + 1)g'(f(\log 2)) = 1$$
$$\Rightarrow (e^{\log 2} + 1)g'(f(\log 2)) = 1$$
$$\Rightarrow g'(f(\log 2)) = \frac{1}{3}$$

2

Derivatives of Functions in Parametric Forms

$x_1 + x_2 = 0$

$x_1 = -x_2$

$\boxed{\begin{matrix} x_2 = t \\ x_1 = -t \end{matrix}}$

$t =$

If

$x = f(t),\ y = g(t)$

$x = \sin t$

$y = \cos t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} =$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$\frac{dy}{}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} =$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \left(\text{whenever } \frac{dx}{dt} \neq 0 \right)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at^3}{4at} = t^2 \qquad \frac{dx}{dy} = \frac{\frac{dx}{dt}}{\frac{dy}{dt}} =$$

$x = 2at^2$

$y = at^4$

1. $x = 2at^2,\ y = at^4$
2. $x = a \cos \theta,\ y = b \cos \theta$
3. $x = \sin t,\ y = \cos 2t$
4. $x = 4t,\ y = \dfrac{4}{t}$
5. $x = \cos \theta - \cos 2\theta,\ y = \sin \theta - \sin 2\theta$
6. $x = a\,(\theta - \sin \theta),\ y = a\,(1 + \cos \theta)$
7. $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}},\ y = \dfrac{\cos^3 t}{\sqrt{\cos 2t}}$
8. $x = a\left(\cos t + \log \tan \dfrac{t}{2}\right)\ y = a \sin t$
9. $x = a \sec \theta,\ y = b \tan \theta$
10. $x = a\,(\cos \theta + \theta \sin \theta),\ y = a\,(\sin \theta - \theta \cos \theta)$
11. If $x = \sqrt{a^{\sin^{-1} t}},\ y = \sqrt{a^{\cos^{-1} t}}$, show that $\boxed{\dfrac{dy}{dx} = -\dfrac{y}{x}}$

$xy = \sqrt{a^{\sin^{-1}t + \cos^{-1}t}} = \sqrt{a^{\pi/2}}$

$y = \sqrt{a^{\cos^{-1}t}}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2\sqrt{a^{\cos^{-1}t}}} \cdot \frac{-\log a \cdot a^{\cos^{-1}t}}{\sqrt{1-t^2}}}{\frac{1}{2\sqrt{a^{\sin^{-1}t}}} \cdot \frac{\log a \cdot a^{\sin^{-1}t}}{\sqrt{1-t^2}}}$$

$$\frac{dy}{dt} = \frac{1}{2\sqrt{a^{\cos^{-1}t}}} \cdot \frac{d}{dt}\left(a^{\cos^{-1}t}\right)$$

$x = \sqrt{a^{\sin^{-1}t}}$

$v = a^{\cos^{-1}t}$

$\log v = \cos^{-1}t \cdot \log a$

$$\frac{1}{v}\frac{dv}{dt} = \log a \cdot \frac{-1}{\sqrt{1-t^2}}$$

$$\frac{dv}{dt} = a^{\cos^{-1}t} \cdot \frac{\log a\,(-1)}{\sqrt{1-t^2}}$$

$$\boxed{\frac{dy}{dx} = -\frac{\sqrt{a^{\cos^{-1}t}}}{\sqrt{a^{\sin^{-1}t}}} = \frac{-y}{x}}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$y = f(x)$

$y' = \frac{d}{dx}(f(x))$

$y'' = \frac{d^2}{dx^2}(f(x)) = \frac{d^2 f}{dx^2}$

Second Order Derivative

$y = x^{20} \quad y' = 20x^{19} \quad y'' = 20 \cdot 19\, x^{18} = 380 x^{18}$

Find the second order derivatives of the functions given in Exercises 1 to 10.

$y = x^{20}$ $\quad y' = 20x^{19}$ $\quad y'' = 20\cdot 19\,x^{18} = 380x^{18}$

Find the second order derivatives of the functions given in Exercises 1 to 10.

1. $x^2 + 3x + 2$ 2. x^{20} 3. $x \cdot \cos x$
4. $\log x$ 5. $x^3 \log x$ 6. $e^x \sin 5x$
7. $e^{6x} \cos 3x$ 8. $\tan^{-1} x$ 9. $\log (\log x)$
10. $\sin (\log x)$
11. If $y = 5 \cos x - 3 \sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

$y = x^2+3x+2$, $\quad y' = 2x+3$, $\quad y'' = 2$

$y = x\cdot\cos x$, $\quad y' = x\cdot(-\sin x) + \cos x \cdot 1$, $\quad y'' = -x\cos x - \sin x - \sin x$

$y' = -5\sin x - 3\cos x$, $\quad y'' = -5\cos x + 3\sin x$, $\quad \frac{d^2y}{dx^2} + y = 0$

Q3. $x = t\cos t$, $y = t + \sin t$, then $\boxed{\frac{d^2x}{dy^2}}$ at $t = \frac{\pi}{2}$ is

A. $\frac{\pi+4}{2}$

B. $-\frac{\pi+4}{2}$

C. -2

D. None of these

$\frac{d^2x}{dy^2} =$

$x = t\cos t$, $\quad \frac{dx}{dt} = t(-\sin t) + \cos t$

$y = t + \sin t$, $\quad \boxed{\frac{dy}{dt} = 1 + \cos t}$, $\quad \frac{dt}{dy} = \boxed{\frac{1}{1+\cos t}}$

$\frac{dx}{dy} = \frac{-t\sin t + \cos t}{1+\cos t}$

$\frac{d^2x}{dy^2} = \frac{d}{dt}\left(\frac{dx}{dy}\right)\cdot\frac{dt}{dy}$

$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{\frac{d}{dt}\left(\frac{dx}{dy}\right)}{\frac{dy}{dt}} = \frac{d}{dt}\left(\frac{dx}{dy}\right)\boxed{\frac{dt}{dy}}$

$= \frac{(1+\cos t)(-t\cos t - \sin t - \sin t) - (-t\sin t + \cos t)(-\sin t)}{(1+\cos t)^2} \cdot \frac{1}{1+\cos t}$

$\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$

$= \frac{(1)(0-1-1) - (-\frac{\pi}{2}+0)(-1)}{(1+0)^3} = -2 - \frac{\pi}{2} = \frac{-4-\pi}{2} = -\left(\frac{4+\pi}{2}\right)$

If $x = 3\tan t$ and $y = 3\sec t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$, is:

A. $\frac{3}{2\sqrt{2}}$

B. $\frac{1}{3\sqrt{2}}$

C. $\frac{1}{6}$

D. $\frac{1}{6\sqrt{2}}$

$x = 3\tan t$, $\quad \frac{dx}{dt} = 3\sec^2 t$, $\quad \frac{dt}{dx} = \frac{1}{3\sec^2 t}$

$y = 3\sec t$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3\sec t\tan t}{3\sec^2 t}$

$\frac{dy}{dx} = \tan t\cdot\cos t = \frac{\sin t}{\cos t}\cdot\cos t = \sin t$

$\boxed{\frac{dy}{dx} = \sin t}$

$\frac{d}{dt}\left(\frac{dy}{dx}\right) \neq \frac{d^2y}{dt\,dx}$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\boxed{\frac{dt}{dx}}$

$\boxed{\frac{d^2y}{dx^2}}$

$= \cos t \cdot \frac{1}{3\sec^2 t} = \frac{1}{3}\cos^3 t = \frac{1}{3}\left(\cos\frac{\pi}{4}\right)^3 = \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{3}\cdot\frac{1}{2\sqrt{2}}$

Q10. If u = x^2 + y^2 and x = s + 3t, y = 2s - t, then $\frac{d^2u}{ds^2}$ equals to

A 12

B 32

C 36

D 10

$u = x^2 + y^2$

$\frac{du}{ds} = 2x\frac{dx}{ds} + 2y\frac{dy}{ds}$

$\frac{d^2u}{ds^2} = 2\left(\frac{dx}{ds}\right)^2 + 2x\frac{d^2x}{ds^2} + 2\left(\frac{dy}{ds}\right)^2 + 2y\frac{d^2y}{ds^2}$

$\frac{dx}{ds} = 1$ $\frac{dy}{ds} = 2$

$\frac{d^2x}{ds^2} = 0$ $\frac{d^2y}{ds^2} = 0$

$\frac{du}{dx} = 2x$

$\frac{du}{ds} = 2x\frac{dx}{ds} + 2y\cdot\frac{dy}{ds}$

$\frac{d^2u}{ds^2} = 2x\cdot\frac{d^2x}{ds^2} + 2\left(\frac{dx}{ds}\right)^2 + 2y\cdot\frac{d^2y}{ds^2} + 2\left(\frac{dy}{ds}\right)^2$

$= 2x\cdot\frac{d^2x}{ds^2} + 2\left(\frac{dx}{ds}\right)^2$

$\frac{d^2u}{ds^2} = 2(1)^2 + 2x(0) + 2(2)^2 + 2y\cdot(0)$

$= 2 + 8 = 10$

$y = (f(x))^2$

$y' = 2\cdot f(x)\cdot f'(x)$

$= \frac{1}{3}\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{3}\frac{1}{2\sqrt{2}}$

$= \frac{1}{6\sqrt{2}}$

$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \sin t$

$\frac{d}{dt}\left(\frac{dy}{dx}\right)\cdot\frac{dt}{dx}$

$\frac{d}{dx^2}(y)$

Unit: 2P2

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Integration

$\frac{d}{dx}(x^2) = \boxed{2x}$

Integration is the inverse process of differentiation. Instead of differentiating a function, we are given the derivative of a function and asked to find its primitive, i.e., the original function. Such a process is called integration or anti differentiation.

$\int 2x = \boxed{x^2 + C}$

$f(x) = x^2 \qquad x^2+1 \qquad x^2+2 \qquad x^2+c$

$f'(x) = 2x \qquad 2x \qquad 2x$

Derivatives	Integrals (Anti derivatives)
(i) $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$;	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$
Particularly, we note that $\frac{d}{dx}(x) = 1$;	$\int dx = x + C$
(ii) $\frac{d}{dx}(\sin x) = \cos x$;	$\int \cos x\,dx = \sin x + C$
(iii) $\frac{d}{dx}(-\cos x) = \sin x$;	$\int \sin x\,dx = -\cos x + C$
(iv) $\frac{d}{dx}(\tan x) = \sec^2 x$;	$\int \sec^2 x\,dx = \tan x + C$
(v) $\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$;	$\int \operatorname{cosec}^2 x\,dx = -\cot x + C$
(vi) $\frac{d}{dx}(\sec x) = \sec x \tan x$;	$\int \sec x \tan x\,dx = \sec x + C$
(vii) $\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$;	$\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + C$
(viii) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
(ix) $\frac{d}{dx}(-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$
(x) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$;	$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
(xi) $\frac{d}{dx}(-\cot^{-1} x) = \frac{1}{1+x^2}$;	$\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$
(xii) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$;	$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$
(xiii) $\frac{d}{dx}(-\operatorname{cosec}^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$;	$\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$
(xiv) $\frac{d}{dx}(e^x) = e^x$;	$\int e^x\,dx = e^x + C$
(xv) $\frac{d}{dx}(\log\|x\|) = \frac{1}{x}$;	$\int \frac{1}{x}\,dx = \log\|x\| + C$
(xvi) $\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$;	$\int a^x\,dx = \frac{a^x}{\log a} + C$

$\sin x + 2$

$f(x) = \sin x + 5$

$f'(x) = \cos x + 0$

$\boxed{\sin x + C}$

0

$\frac{d}{dx}(5) = 0$

Some properties of indefinite integral

(a)The process of differentiation and integration are inverses of each other in the sense of the following results

$\frac{d}{dx}\int f(x)\,dx = f(x)$

$\int f'(x)\,dx = f(x) + C$, where C is any arbitrary constant.

$\frac{d}{dx}\int f(x) = f(x)$

$\int \frac{d}{dx} f(x) = f(x) + c$

(b)Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent.

$(f(x))' = \boxed{g(x)}$

©

$(u \pm v)' = u' \pm v'$

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$\int f_1 + f_2 + \dots + f_n \,dx = \int f_1\,dx + \int f_2\,dx + \dots + \int f_n\,dx$

(d)

For any real number k, $\int k\, f(x)\,dx = k \int f(x)\,dx$

$\int \sin x \cdot \log a \,dx = \log a \int \sin x\,dx$

Ex.Find

(i) $\cos 2x$ (ii) $3x^2 + 4x^3$ (iii) $\frac{1}{x}, x \neq 0$

Ex.Find

$\boxed{\frac{d}{dx}(\log x) = \frac{1}{x}}$

$\int x^{-1+1} \over -1+1$ $\left(\int \frac{1}{0}\right) = \infty$

(i) $\int \frac{x^3 - 1}{x^2}\,dx$ (ii) $\int (x^{\frac{2}{3}} + 1)\,dx$ (iii) $\int (x^{\frac{3}{2}} + 2\,e^x - \frac{1}{x})\,dx$

$\int \left(x - \frac{1}{x^2}\right)dx = \int x - \int x^{-2}$

$= \frac{x^{1+1}}{1+1} - \frac{x^{-2+1}}{-2+1} + c$

$= \frac{1}{2}x^2 + x^{-1} + c$

$\frac{x^{2/3+1}}{\frac{2}{3}+1} + x$ | $\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 2e^x - \log x + c$

$\int \tan x \sec x = \sec x$

Ex. Find

$= \frac{1}{2}x^2 + x^{-1} + c$

$\int \tan \sec x = \sec x$

Ex. Find

(i) $\int (\sin x + \cos x)\, dx$ (ii) $\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x)\, dx$

(iii) $\int \frac{1 - \sin x}{\cos^2 x}\, dx$

$-\cos x + \sin x + c$

$\int \operatorname{cosec}^2 x + \operatorname{cosec} x \cot x$

$-\cot x - \operatorname{cosec} x + c$

$\int \frac{1 - \sin x}{\cos^2 x} = \int \sec^2 x - \int \tan x \cdot \sec x$

$= \tan x - \sec x + c$

Ex. Find

6. $\int (4e^{3x} + 1)\, dx$ 7. $\int x^2 (1 - \frac{1}{x^2})\, dx$ 8. $\int (ax^2 + bx + c)\, dx$

$\frac{4e^{3x}}{3} + x + c$ $\frac{ax^3}{3} + \frac{bx^2}{2} + cx + c'$

9. $\int (2x^2 + e^x)\, dx$ 10. $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 dx$ 11. $\int \frac{x^3 + 5x^2 - 4}{x^2}\, dx$

12. $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$ 13. $\int \frac{x^3 - x^2 + x - 1}{x - 1}\, dx$ 14. $\int (1 - x)\sqrt{x}\, dx$ $= \int \sqrt{x} - x^{3/2}\, dx$

$= \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c$

15. $\int \sqrt{x}(3x^2 + 2x + 3)\, dx$ 16. $\int (2x - 3\cos x + e^x)\, dx$

17. $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$ 18. $\int \sec x (\sec x + \tan x)\, dx$

$\sec^2\theta - \tan^2\theta = 1$

19. $\int \frac{\sec^2 x}{\operatorname{cosec}^2 x}\, dx$ 20. $\int \frac{2 - 3\sin x}{\cos^2 x}\, dx.$

$\int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}} = \int \frac{\sin^2 x}{\cos^2 x} = \int \tan^2 x = \int (\sec^2 x - 1)\, dx$

$= \boxed{\tan x - x + c}$

Integration by substitution

Ex. Find

(i) $\sin mx$ (ii) $2x \sin (x^2 + 1)$

$x^2 + 1 = t$

$2x\, dx = dt$

$\boxed{-\cos(x^2+1) + c}$

(iii) $\frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$ (iv) $\frac{\sin (\tan^{-1} x)}{1 + x^2}$

$= \frac{1}{1+x^2}\, dx = dt$

$= -\cos(\tan^{-1} x) + c$

$m\, dx = dt$

$mx = t$

$m \cdot 1 = \frac{dt}{dx}$

$\int \sin mx\, dx$

$\int \sin t \cdot \frac{dt}{m}$

$\frac{1}{m} \int \sin t\, dt = \frac{1}{m}(-\cos t) + c$

$= -\frac{1}{m} \cos(mx) + c$

$\int 2x \sin(x^2+1)\, dx$

$\frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}$

Put $\tan \sqrt{x} = t$

$\sqrt{x} = t$

$= 2\frac{\tan^5 \sqrt{x}}{5} + c$

Ex. Find

(i) $\int \sin^3 x \cos^2 x \, dx$ (ii) $\int \frac{\sin x}{\sin(x+a)} dx$ (iii) $\int \frac{1}{1+\tan x} dx$

$\int \sin^2 x . \cos^2 x . \sin x \, dx$

$\int (1-\cos^2 x) . \cos^2 x . \sin x \, dx = \int (1-t^2)(t^2) . dt$

$\cos x = t$

$-\sin x \, dx = dt$

$\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$

$x+a = t$

$dx = dt$

$\int \frac{\sin x}{\sin(x+a)} . dx = \int \frac{\sin(t-a)}{\sin t} . dt$

$= \int \frac{\sin(t)\cos a - \cos(t)\sin a}{\sin t} dt$

$\int \frac{f'(x)}{f(x)} = \log|f(x)|$

$= \int (\cos a - \sin a . \cot t) dt$

$= \cos a . t - \sin a . \log|\sin t| + C$

$\int \frac{\cos(t)}{\sin(t)} = \log|\sin t|$

$= \cos a \, (x+a) - \sin a \log|\sin(x+a)| + C$

$1+\tan x = t$

$\sec^2 x \, dx = dt$

$u = \tan x$

$\int \frac{1}{1+\tan x} dx = \int \frac{1}{1+\frac{\sin x}{\cos x}} dx = \int \frac{\cos x}{\cos x + \sin x} dx$

$= \frac{1}{2}\int \frac{2\cos x}{\cos x + \sin x}$

$\int \frac{2x \, dx}{x^2+1} = \int \frac{f'(x)}{f(x)} = \log|f(x)|$

$= \frac{1}{2}\int \frac{\cos x + \sin x + \cos x - \sin x}{\cos x + \sin x}$

$= \frac{1}{2}\int 1 + \frac{\cos x - \sin x}{\cos x + \sin x}$

$= \frac{1}{2}(x + \log(\cos x + \sin x)) + C$

Ex. Find

$\log|1+x^2| + C$

1. $\int \frac{2x}{1+x^2} dx$ 2. $\frac{(\log x)^2}{x}$ 3. $\frac{1}{x + x\log x}$

4. $\sin x \sin(\cos x)$ 5. $\sin(ax+b)\cos(ax+b)$

$1 + \log x = t$

$0 + \frac{1}{x} dx = dt$

$\int \frac{1}{x(1+\log x)} dx = \int \frac{1}{t} dt = \log(t) + C$

$= \log(1+\log x) + C$

6. $\sqrt{ax+b}$

7. $x\sqrt{x+2}$

8. $x\sqrt{1+2x^2}$

9. $(4x+2)\sqrt{x^2+x+1}$

10. $\dfrac{1}{x-\sqrt{x}}$

11. $\dfrac{x}{\sqrt{x+4}}$, $x>0$

12. $(x^3-1)^{\frac{1}{3}}\,x^5$

13. $\dfrac{x^2}{(2+3x^3)^3}$

14. $\dfrac{1}{x\,(\log x)^m}$, $x>0$, $m\neq 1$

15. $\dfrac{x}{9-4x^2}$

16. e^{2x+3}

17. $\dfrac{x}{e^{x^2}}$

18. $\dfrac{e^{\tan^{-1}x}}{1+x^2}$

19. $\dfrac{e^{2x}-1}{e^{2x}+1}$

20. $\dfrac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}$

21. $\tan^2(2x-3)$

22. $\sec^2(7-4x)$

23. $\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$, $\sin^2 x = \dfrac{1-\cos 2x}{2}$ $\qquad$ Sin A . Cos B

$\sin 3x = 3\sin x - 4\sin^3 x \Rightarrow \sin^3 x = \dfrac{\sin 3x - 3\sin x}{-4}$

Integration using trigonometric identities

Find (i) $\int \cos^2 x\,dx$ (ii) $\int \sin 2x \cos 3x\,dx$ (iii) $\int \sin^3 x\,dx$

$\sin(-\theta) = -\sin\theta$

$\frac{1}{2}\int (1+\cos 2x)\,dx = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$

$\int \sin 2x \cos 3x = \int \frac{1}{2}\left(\sin(5x) - \sin(x)\right) = \frac{1}{2}\left(\frac{-\cos 5x}{5}\right) + \frac{1}{2}\cos x + c$

$\int \sin^3 x\,dx = \frac{-1}{4}\int (\sin 3x - 3\sin x)\,dx = -\frac{1}{4}\left[\frac{-\cos 3x}{3} + 3\frac{\cos x}{1}\right] + c$

1. $\sin^2(2x+5)$ 2. $\sin 3x \cos 4x$ 3. $\cos 2x \cos 4x \cos 6x$

4. $\sin^3(2x+1)$ 5. $\sin^3 x \cos^3 x$ 6. $\sin x \sin 2x \sin 3x$

7. $\sin 4x \sin 8x$ 8. $\dfrac{1-\cos x}{1+\cos x}$ 9. $\dfrac{\cos x}{1+\cos x}$

10. $\sin^4 x$ 11. $\cos^4 2x$ 12. $\dfrac{\sin^2 x}{1+\cos x}$

13. $\dfrac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}$ 14. $\dfrac{\cos x - \sin x}{1+\sin 2x}$ 15. $\tan^3 2x \sec 2x$

16. $\tan^4 x$ 17. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$ 18. $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

19. $\dfrac{1}{\sin x \cos^3 x}$ 20. $\dfrac{\cos 2x}{(\cos x + \sin x)^2}$ 21. $\sin^{-1}(\cos x)$

Integrals of Some Particular Functions

(1) $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + C$

(2) $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + C$

(3) $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + C$

(4) $\int \dfrac{dx}{\sqrt{x^2-a^2}} = \log\left|x+\sqrt{x^2-a^2}\right| + C$

(5) $\int \dfrac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\dfrac{x}{a} + C$

(6) $\int \dfrac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$

ax^2+bx+c

$x^2+\frac{b}{a}x+\frac{c}{a}$

$x^2+\frac{b}{a}x+\frac{c}{a}+\left(\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2$

Ex. Find

(i) $\int \dfrac{dx}{x^2-16}$ (ii) $\int \dfrac{dx}{\sqrt{2x-x^2}}$

$\int \dfrac{dx}{x^2-(4)^2} = \dfrac{1}{8}\log\left|\dfrac{x-4}{x+4}\right| + c$

$\int \dfrac{dx}{\sqrt{-(x^2-2x)}}$ $\left(\frac{2}{2}\right)^2 = 1$

$= \int \dfrac{dx}{\sqrt{-(x^2-2x+1-1)}}$

$= \int \dfrac{dx}{\sqrt{1-(x-1)^2}}$

$= \sin^{-1}\left(\dfrac{x-1}{1}\right) + c$

(i) $\int \dfrac{dx}{x^2-6x+13}$ (ii) $\int \dfrac{dx}{3x^2+13x-10}$ (iii) $\int \dfrac{dx}{\sqrt{5x^2-2x}}$

$\int \dfrac{dx}{x^2-6x+13+9-9} = \int \dfrac{dx}{(x-3)^2+(2)^2}$

$= \dfrac{1}{2}\tan^{-1}\left(\dfrac{x-3}{2}\right) + c$

$\int \dfrac{dx}{ax^2+bx+c}$

(i) $\int \dfrac{x+2}{2x^2+6x+5}dx$ (ii) $\int \dfrac{x+3}{\sqrt{5-4x+x^2}}dx$

$2x^2+6x+5$

$4x+6$

$\int \frac{f'(x)}{f(x)} = \ln f(x) + c$

$\frac{1}{4}\int \dfrac{4x+8}{2x^2+6x+5}dx = \frac{1}{4}\int \dfrac{4x+6}{\ldots} + \frac{1}{4}\int \dfrac{2}{\ldots}$ $\left(\frac{3}{2}\right)^2$

$\int \frac{f'(x)}{f(x)} = \log f(x) + c$

$\frac{1}{4}\int \frac{4x+8}{2x^2+6x+5}\,dx = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5} + \frac{1}{4}\int \frac{2}{2x^2+6x+5}$

$\frac{1}{4}\log|2x^2+6x+5| + \frac{1}{2}\int \frac{dx}{x^2+3x+\frac{5}{2}+\frac{9}{4}-\frac{9}{4}}$

x^2+bx+c

$x^2+bx+c+\left(\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2$

$\frac{1}{4}\log|2x^2+6x+5| + \frac{1}{4}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}$

$= \frac{1}{4}\log|2x^2+6x+5| + \frac{1}{4}\cdot\frac{2}{1}\tan^{-1}\left(\frac{x+\frac{3}{2}}{\frac{1}{2}}\right)+c$

$\frac{(x^2+1)\,dx}{x^3+4x+c}$

$\frac{1}{2\sqrt{x^2-1}}\,2x\,dx = dt$

1. $\frac{3x^2}{x^6+1}$ 2. $\frac{1}{\sqrt{1+4x^2}}$ 3. $\frac{1}{\sqrt{(2-x)^2+1}}$

4. $\frac{1}{\sqrt{9-25x^2}}$ 5. $\frac{3x}{1+2x^4}$ 6. $\frac{x^2}{1-x^6}$

7. $\frac{x-1}{\sqrt{x^2-1}}$ 8. $\frac{x^2}{\sqrt{x^6+a^6}}$ 9. $\frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

x^2-1

$2x$

$\frac{dt}{\sqrt{t}}$

$\sqrt{x^2-1}=t$

$x^3=t$

$3x^2\,dx=dt$

$\frac{1}{2}\int\frac{2x}{\sqrt{x^2-1}} - \int\frac{1}{\sqrt{x^2-1}}$

$\int\frac{x^2}{\sqrt{x^6+a^6}} = \int\frac{dt}{3\sqrt{x^2+(a^3)^2}}$

Integration by Partial Fractions

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}$, $a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$,
	where x^2+bx+c cannot be factorised further	

$\int\frac{px+q}{(x-a)(x-b)}$

$=\int\frac{A}{x-a}+\int\frac{B}{x-b}$

$=A\log(x-a)+B\log(x-b)$

x^2+1 $x=\pm i$

$(x-1)(x+1)$

x^2+bx+c

$\int\frac{x}{(x+1)(x+2)}$

$L.H.S = R.H.S$

1. $\frac{x}{(x+1)(x+2)}$ 2. $\frac{1}{x^2-9}$ 3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

$\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)}+\frac{B}{(x+2)}$

$1x+0 = A(x+2)+B(x+1) = Ax+2A+Bx+B = (A+B)x+(2A+B)$

$1x + 0 = A(x+2) + B(x+1) = Ax + 2A + Bx + B = (A+B)x + (2A+B)$

Coff x^1 $\quad 1 = A + B$ — (1)

Coff of x^0 $\quad 0 = 2A + B$ — (2)

$-A = 1$ $\quad$ $\boxed{A = -1}$ $\quad$ $-1 + B = 1$ $\quad$ $\boxed{B = 2}$

$\int \frac{1}{x+1}$

$$\int \frac{x}{(x+1)(x+2)} = \int \frac{-1}{x+1} + \frac{2}{x+2}$$

$$= -1 \log(x+1) + 2\log(x+2) + c$$

$x = -1$

$x = -2$

$$\int \frac{x}{(x+1)(x+2)} = \int \frac{-1}{(x+1)(-1+2)} + \frac{-2}{(-2+1)(x+2)}$$

$$= \int \frac{-1}{x+1} + \frac{2}{x+2}$$

$$= -\log(x+1) + 2\log(x+2) + c$$

$x = -3$

$x = 3$

$$\int \frac{dx}{x^2-9} = \int \frac{dx}{(x-3)(x+3)} = \int \frac{dx}{6(x-3)} + \int \frac{dx}{-6(x+3)}$$

$$= \frac{1}{6}\log(x-3) - \frac{1}{6}\log(x+3) + c$$

$$\int \frac{3x-1}{(x-1)(x-2)(x-3)} = \int \frac{2}{(x-1)2} + \frac{5}{(-1)(x-2)} + \frac{8}{2(x-3)}$$

$$= \log(x-1) - 5\log(x-2) + 4\log(x-3) + c$$

4. $\frac{x}{(x-1)(x-2)(x-3)}$ 5. $\frac{2x}{x^2+3x+2}$ 6. $\frac{1-x^2}{x(1-2x)}$

$$\int \frac{2x}{(x+1)(x+2)}$$

7. $\dfrac{x}{(x^2+1)(x-1)}$ 8. $\dfrac{x}{(x-1)^2(x+2)}$ 9. $\dfrac{3x+5}{x^3-x^2-x+1}$

10. $\dfrac{2x-3}{(x^2-1)(2x+3)}$ 11. $\dfrac{5x}{(x+1)(x^2-4)}$ 12. $\dfrac{x^3+x+1}{x^2-1}$

13. $\dfrac{2}{(1-x)(1+x^2)}$ 14. $\dfrac{3x-1}{(x+2)^2}$ 15. $\dfrac{1}{x^4-1}$

$\int \dfrac{2}{(1-x)(1+x^2)}$

$x^2+1=0$, $x=\pm i$

$\dfrac{2}{(1-x)(x^2+1)} = \dfrac{A}{(1-x)} + \dfrac{Bx+C}{x^2+1}$

$2 = A(x^2+1) + (Bx+C)(1-x) = A(x^2+1) + (Bx - Bx^2 + C - Cx)$

Coff x^2: $0 = A - B$ — (1)

Coff of x: $0 = B - C$ — (2)

Coff x^0: $2 = A + C$ — (3)

(1) + (2): $0 = A - C$

$2 = A + C$

$2A = 2$, $A = 1$, $C = 1$, $B = 1$

$\int \dfrac{2}{(1-x)(x^2+1)} = \int \dfrac{1}{1-x} + \int \dfrac{x+1}{x^2+1}$

$= -\log(1-x) + \frac{1}{2}\log(x^2+1) + \tan^{-1}x + C$

$\frac{1}{2}\int \dfrac{2x}{x^2+1}$

$\int \dfrac{f'(x)}{f(x)}$

$\int \dfrac{1}{x^2+1} = \tan^{-1}x$

Integration by Parts

Sin x Cos x

ILATE

A.T

$$\int f(x)\,g(x)\,dx = f(x)\int g(x)\,dx - \int [f'(x)\int g(x)\,dx]\,dx$$

x, x^2+1, x^3+

e^x, e^{2x}

ILATE

1. $x \sin x$ 2. $x \sin 3x$ 3. $x^2 e^x$ 4. $x \log x$

$\int x \cdot \sin x = x \cdot (-\cos x) - \int 1 \cdot (-\cos x)$

$= -x\cos x + \sin x + C$

$\int \log x \cdot x = \log x \cdot \dfrac{x^2}{2} - \int \dfrac{1}{x}\cdot\dfrac{x^2}{2}$

$= \log x \cdot x^2/2 - \frac{1}{2}\dfrac{x^2}{2}$ $\quad \log x \dfrac{x^2}{2} - \dfrac{x^2}{4} + C$

ILATE

UV

x, x^2, x^3, x^3+1

$\int uv = uv_1 - u'v_2 + u''v_3 - u'''v_4 + u^{iv}v_5 - \cdots$

$\int x \sin x = x\cdot(-\cos x) - 1\cdot(-\sin x)$

$\int x \sin x = x \cdot (-\cos x) - 1 \cdot (-\sin x)$

$\int x^3 \cdot \sin x = x^3(-\cos x) - 3x^2(-\sin x) + 6x(\cos x) - 6(\sin x)$

$\int x^4 \cdot e^x = x^4 e^x - 4x^3 e^x + 12x^2 e^x - 24x e^x + 24 e^x$

$\int x^4 \cdot e^x = x^4 \cdot e^x - \int 4x^3 e^x$

5. $x \log 2x$ 6. $x^2 \log x$ 7. $x \sin^{-1} x$ 8. $x \tan^{-1} x$

9. $x \cos^{-1} x$ 10. $(\sin^{-1} x)^2$ 11. $\dfrac{x \cos^{-1} x}{\sqrt{1-x^2}}$ 12. $x \sec^2 x$

13. $\tan^{-1} x$ 14. $x (\log x)^2$ 15. $(x^2 + 1) \log x$

$\int \frac{1}{\sqrt{x^2+2}}$

Some other formulas

$$\int \sqrt{x^2 - a^2}\, dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2} \log \left| x + \sqrt{x^2 - a^2} \right| + C$$

(ii) $$\int \sqrt{x^2 + a^2}\, dx = \frac{1}{2} x \sqrt{x^2 + a^2} + \frac{a^2}{2} \log \left| x + \sqrt{x^2 + a^2} \right| + C$$

(iii) $$\int \sqrt{a^2 - x^2}\, dx = \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + C$$

$\int \sqrt{x^2+4} = \frac{x}{2}\sqrt{x^2+4} + \frac{4}{2}\log(x + \sqrt{x^2+4}) + C$

$\int dx$

1. $\sqrt{4-x^2}$ 2. $\sqrt{1-4x^2}$ 3. $\sqrt{x^2+4x+6}$
4. $\sqrt{x^2+4x+1}$ 5. $\sqrt{1-4x-x^2}$ 6. $\sqrt{x^2+4x-5}$
7. $\sqrt{1+3x-x^2}$ 8. $\sqrt{x^2+3x}$ 9. $\sqrt{1+\frac{x^2}{9}}$

$\to \sqrt{x^2+4x+6}$

$\sqrt{x^2+4x+6+4-4}$

$= \sqrt{(x+2)^2 - (\sqrt{2})^2}$

Q1. $\int \sec^{\frac{4}{9}} \theta \operatorname{cosec}^{\frac{14}{9}} \theta\, d\theta$ is equal to

$\sec^{4/9}\theta \cdot \operatorname{cosec}^{14/9}\theta\, d\theta$

$d\theta$

$\cos^{4/9}\theta \cdot \sin^{14/9}\theta$

$d\theta$

$\cos^{4/9}\theta \cdot \sin^{14/9}\theta \cdot \cos^{14/9}\theta$

$\cos^{14/9}\theta$

$\sec^2 dt$

$\cos^{4/9}$ $\cos^{14/9}$

$\cos^{4/9+14/9}$

$\cos^{18/9}$

A. $\frac{5}{9}(\tan\theta)^{\frac{-5}{9}} + c$

B. $-\frac{9}{5}(\tan\theta)^{\frac{-5}{9}} + c$

C. $\frac{9}{5}(\tan\theta)^{\frac{-9}{5}} + c$

D. $-\frac{5}{9}(\tan\theta)^{\frac{-9}{5}} + c$

D $-\frac{5}{9}(\tan\theta)^{\frac{-9}{5}}+c$

$\tan\theta = t$

$\sec^2\theta\, d\theta = dt$

$\int t^n = \frac{t^{n+1}}{n+1}$

$\int \frac{\sec^2 dt}{\sec^2\theta\,(\tan\theta)^{14/9}}$

$\int \frac{dt}{t^{14/9}} = \int t^{-14/9}\, dt$

$= \frac{t^{-14/9+1}}{-14/9+1} = \frac{-9}{5} t^{\frac{-5}{9}} + c$

Q2. If $I=\int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt=\frac{1}{2}(g(t))^2+c$, where C is a constant, then g(2) is equal to

A $2\log\left(2+\sqrt{5}\right)$

B $\log\left(2+\sqrt{5}\right)$

C $\frac{1}{\sqrt{5}}\log\left(2+\sqrt{5}\right)$

D $\frac{1}{2}\log\left(2+\sqrt{5}\right)$

$g(t) \cdot g'(t)$

$\log(a+b)$

$\int u\, du = \frac{u^2}{2}+c$

$\log\left(t+\sqrt{1+t^2}\right) = u$

$\frac{1}{t+\sqrt{1+t^2}}\left(1+\frac{2t}{2\sqrt{1+t^2}}\right)dt = du$

$\frac{1}{t+\sqrt{1+t^2}}\frac{\left(\sqrt{1+t^2}+t\right)}{\sqrt{1+t^2}}dt = du$

$\frac{dt}{\sqrt{1+t^2}} = du$

$\log\left(t+\sqrt{1+t^2}\right) = g(t)$

$\log\left(2+\sqrt{1+t^2}\right)$

$\log(2+\sqrt{5})$

Q4. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to

A $(x-1)e^{x+\frac{1}{x}}+c$

B $x\,e^{x+\frac{1}{x}}+c$

C $(x+1)\,e^{x+\frac{1}{x}}+c$

D $-x\,e^{x+\frac{1}{x}}+c$

$\int\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}$

Put $x+\frac{1}{x}=t$

$1-\frac{1}{x^2}dx = dt$

$\int e^t\, dt = e^t = e^{x+\frac{1}{x}}$

$+\int x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}$

$e^{x+\frac{1}{x}} + x\left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}$

$= \int e^{x+\frac{1}{x}} + x \cdot e^{x+\frac{1}{x}} - \int\int\left(1-\frac{1}{x^2}\right)\cdot e^{x+\frac{1}{x}}$

$= \int e^{x+\frac{1}{x}} + x\cdot e^{x+\frac{1}{x}} - \int e^{x+\frac{1}{x}} + c$

$x\cdot e^{x+\frac{1}{x}} + c$

$\int \frac{1}{x} = \log(x)$

$\int \frac{1}{x-1} = \log(x-1)$

$\int \frac{1}{x^3-1} \neq \log(x^3-1)$

Q5. If $\int f(x)dx = \psi(x)$, then $\int x^5 f(x^3)\, dx$ is equal to

$x^3 = t$

$3x^2 \cdot dx = dt$

$x^5 \cdot \int f(x^5)$

A $\frac{1}{3}x^3\psi(x^3) - 3\int x^3\psi(x^3)dx + C$

B $\frac{1}{3}x^3\psi(x^3) - \int x^3\psi(x^3)dx + C$

C $\frac{1}{3}\left[x^3\psi(x^3) - \int x^3\psi(x^3)dx\right] + C$

D $\frac{1}{3}\left[x^3\psi(x^3) - \int x^2\psi(x^3)dx\right] + C$

$\int x^3 \cdot f(x^3) \cdot x^2 dx$

$\int t \cdot f(t) \cdot dt$

$\frac{1}{3}\left(t \cdot \psi(t) - \int \psi(t) \cdot dt\right)$

$\frac{1}{3}\left(x^3 \psi(x^3) - \int \psi(x^3)\, 3x^2\right) + C$

Q7. If the integral $\int \frac{5\tan x}{\tan x - 2}dx = x + a\log|\sin x - 2\cos x| + k$, then a is equal to

$\int 1\, dx = x$

A -2

B 1

C 2

D -1

$\int \frac{\frac{5\sin x}{\cos x}}{\frac{\sin x}{\cos x} - 2} = \int \frac{5\sin x \, dx}{\sin x - 2\cos x}$

$= \int \frac{\sin x - 2\cos x + 4\sin x + 2\cos x}{\sin x - 2\cos x}$

$= \int \frac{\sin x - 2\cos x}{\sin x - 2\cos x} + 2\int \frac{2\sin x + \cos x}{\sin x - 2\cos x}$

$= x + 2 \cdot \log|\sin x - 2\cos x|$

$\int \frac{f'(x)}{f(x)}$

$\int \frac{\sin x}{\sin(a+x)}$

Q. The value of $\sqrt{2}\int \frac{\sin x dx}{\sin\left(x - \frac{\pi}{4}\right)}$ is

$\sqrt{2}\int \sin\left(x - \frac{\pi}{4} + \frac{\pi}{4}\right)$

$\left(x - \frac{\pi}{4}\right)$

$\sin(A+B)$

$\sin(A)\cos B + \cos A \sin B$

A $x + \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

B $x - \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

C $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

$\sqrt{2}\, \sin\left(x - \frac{\pi}{4}\right) \cdot \cos\frac{\pi}{4} + \cos\left(x - \frac{\pi}{4}\right) \cdot \frac{1}{\sqrt{2}}$

$\sin\left(x - \frac{\pi}{4}\right)$

$\sin\left(x - \frac{\pi}{4}\right)$

$\int \cot \cdot \int \cos$

$\sqrt{2}\int$

C $x + \log\left|\sin\left(x - \frac{\pi}{4}\right)\right| + c$

D $x - \log\left|\cos\left(x - \frac{\pi}{4}\right)\right| + c$

$\int \cot = \int \frac{\cos}{\sin}$, $\log(\sin x)$

$\sin\left(x - \frac{\pi}{4}\right)$

$\sqrt{2}\int \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cot\left(x - \frac{\pi}{4}\right)$

$x + \log\left(\sin\left(x - \frac{\pi}{4}\right)\right) + c$

Q6. Evaluate : $\int \frac{x^2\,dx}{(x\sin x + \cos x)^2}$

Q6. The integral $\int \frac{dx}{(x+4)^{\frac{8}{7}}(x-3)^{\frac{6}{7}}}$ is equal to :

(where C is a constant of integration)

A $\left(\frac{x-3}{x+4}\right)^{\frac{1}{7}} + C$

B $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{\frac{-13}{7}} + C$

C $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{\frac{3}{7}} + C$

D $-\left(\frac{x-3}{x+4}\right)^{\frac{-1}{7}} + C$

$(x+4)^{8/7}\cdot\left(\frac{x-3}{x+4}\right)^{6/7}\cdot(x+4)^{6/7}$

$\left(\frac{x-3}{x+4}\right)^{-6/7} dx$

$(x+4)^2$

$\frac{x-3}{x+4} = t$

$\frac{x+4-x+3}{(x+4)^2} = \frac{7}{(x+4)^2}dx = dt$

$\int t^{-6/7}\frac{dt}{7}$

$\frac{t^{-6/7+1}}{-6/7+1} + c$

$= \frac{1}{7}\cdot\frac{7}{1}\cdot t^{1/7} + c$

Q9. If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2}dx, (x \geq 0)$ and $f(0) = 0$ then the value of f (1) is:

A $-\frac{1}{2}$

B $\frac{1}{2}$

C $-\frac{1}{4}$

D $\frac{1}{4}$

$(x^7)^2 = x^{14}$

$\frac{x^2}{x^7}$

$x^{14}\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)^2$

$\int \frac{5x^{-6} + 7x^{-8}}{\left(\frac{1}{x^5} + \frac{1}{x^7} + 2\right)^2}dx$

$(1+x)$, $x\left(\frac{1}{x}+1\right)$

$x^{8-14} = x^{-6}$

$\frac{1}{x^5} + \frac{1}{x^7} + 2 = t$

$-5x^{-6} - 7x^{-8}dx = dt$

$5x^{-6} + 7x^{-8}dx = -dt$

$\int \frac{-dt}{t^2}$

$\frac{1}{t} + c$

$\frac{1}{1+t+t^2}$

$f(x) = \frac{1}{\frac{1}{x} + \frac{1}{x^7} + 2}$

$f(x) = \frac{1}{\frac{1}{x^5} + \frac{1}{x^7} + 2} + c$

$f(0) = 0$, $\frac{1}{\frac{1}{0} + \frac{1}{0} + 2} + c = 0$

$\frac{1}{\infty + \infty + 2} + c = 0$, $c = 0$

$(x^6)^{2/3}$

Q2. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + c$ where C is a constant of integration, then the function

$(x^6)^{2/3}$

Q2. If $\int \frac{dx}{x^3(1+x^6)^{2/3}} = xf(x)(1+x^6)^{\frac{1}{3}} + c$ where C is a constant of integration, then the function f(x) is equal to:

dx $\qquad$ dx

$x^3 \cdot x^4\left(\frac{1}{x^6}+1\right)^{2/3} = x^7\left(\frac{1}{x^6}+1\right)^{2/3}$

A. $-\frac{1}{2x^2}$

B. $-\frac{1}{2x^3}$

C. $\frac{1}{2x^3}$

D. $\frac{3}{x^2}$

$\frac{1}{x^6}+1 = t$

$-6x^{-7}dx = dt$

$\frac{dx}{x^7} = \frac{dt}{-6}$

$\frac{1}{x^6}+1 = \frac{1+x^6}{x^6}$

$-\frac{1}{6}\int \frac{dt}{t^{2/3}} = -\frac{1}{6}\cdot\frac{t^{-2/3+1}}{-\frac{2}{3}+1} + c$

$= -\frac{1}{6}\cdot\frac{3}{1}\cdot t^{1/3} + c$

$-\frac{1}{2}\left(\frac{1}{x^6}+1\right)^{1/3} + c$

$-\frac{1}{2}\frac{(1+x^6)^{1/3}}{(x^6)^{1/3}} + c$

$\frac{x}{x}\cdot -\frac{1}{2}\frac{1}{x^2}(1+x^6)^{1/3} + c$

$= xf(x)(1+x^6)^{1/3} + c$

$f(x) = -\frac{1}{2x^3}$

Q3. If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + c$ for a suitable chosen integer m and a function A(x), where C is constant of integration, than $(A(x))^m$ equals:

Hint = x^2 common from num

A. $\frac{-1}{27x^9}$

B. $\frac{-1}{3x^3}$

C. $\frac{1}{27x^6}$

D. $\frac{1}{9x^4}$

x^4+x

$4x^3+1$

Q4. The integral $\int \frac{2x^3-1}{x^4+x}dx$ equal:

$\frac{4x^3+1-2x^3-2}{x^4+x}$

$f(x)$

$\int \frac{x^3+1}{x(x^3+1)}$

A. $\frac{1}{2}\log_e\frac{|x^3+1|}{x^2} + c$

B. $\frac{1}{2}\log_e\frac{|x^3+1|^2}{|x^3|} + c$

C. $\log_e\left|\frac{x^3+1}{x}\right| + c$

D. $\log_e\frac{|x^3+1|}{x^2} + c$

$\frac{4x^3+1}{x^4+x} - 2\,\frac{x^3+1}{x^4+x}$

$\log(x^4+x) - 2\log x = \log\left(\frac{x^4+x}{x^2}\right) + c$

[redacted]

$\log(x^2+x) - 2\log x = \log\left(\frac{x^2+\ldots}{x^2}\right)+c$

$$\int \frac{\cancel{x^2}\left(2x - \frac{1}{x^2}\right)}{\cancel{x^2}\left(x^2+\frac{1}{x}\right)} = \int \frac{2x - \frac{1}{x^2}}{x^2+\frac{1}{x}} = \log\left(x^2+\frac{1}{x}\right)+c = \log\left(\frac{x^3+1}{x}\right)+c$$

Q1. The integral $\int \frac{2x^{12}+5x^9}{(x^5+x^3+1)^3}dx$ is equal to:

$$= \int \frac{2x^{12}+5x^9}{x^{15}\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3} = \int \frac{2x^{-3}+5x^{-6}}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^3}dx \qquad 1+\frac{1}{x^2}+\frac{1}{x^5} = t$$

$0 - 2x^{-3} - 5x^{-6}$

$$= \int \frac{-dt}{t^3} = -\frac{t^{-3+1}}{-3+1} = \frac{1}{2}t^{-2}+c$$

$$\frac{1}{2}\frac{1}{\left(1+\frac{1}{x^2}+\frac{1}{x^5}\right)^2} \qquad \frac{1}{2}\frac{x^{10}}{(x^5+x^3+1)^2}$$

A $\frac{x^{10}}{2(x^5+x^3+1)^2}+c$

B $\frac{x^5}{2(x^5+x^3+1)^2}+c$

C $\frac{-x^{10}}{2(x^5+x^3+1)^2}+c$

D $\frac{-x^5}{(x^5+x^3+1)^2}+c$

Q2. The integral $\int \frac{dx}{x^2(x^4+1)^{3/4}}$ equal:

A $(x^4+1)^{1/4}+c$

B $-\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$

C $-(x^4+1)^{1/4}+c$

D $\left(\frac{x^4+1}{x^4}\right)^{1/4}+c$

Unit: 2P3

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$$\boxed{\int f(x)\,dx = F(x) + c}$$

$$\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$$

Definite integral

$$\int_a^b f(x)\,dx$$

Ex. Find the integrals

1. $\int_a^b x\,dx$ 2. $\int_0^5 (x+1)\,dx$ 3. $\int_2^3 x^2\,dx$

4. $\int_1^4 (x^2 - x)\,dx$ 5. $\int_{-1}^1 e^x\,dx$ 6. $\int_0^4 (x + e^{2x})\,dx$

$$\left[\frac{x^2}{2}\right]_a^b = \frac{b^2}{2} - \frac{a^2}{2}$$

$$\int_1^4 (x^2 - x)\,dx = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^4 = \frac{(4)^3}{3} - \frac{(4)^2}{2} - \frac{1}{3} + \frac{1}{2}$$

$$5).\ \int_{-1}^1 e^x\,dx = [e^x]_{-1}^1 = e^1 - e^{-1}$$

$\int_1^2 x^2\,dx$

(a) 1

(b) $\frac{7}{3}$

(c) $\frac{1}{3}$

(d) 0

$$\left[\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

$\int_0^2 (x^2 + 3)\,dx$

(a) $\frac{25}{3}$

(b) $\frac{26}{3}$

(c) $\frac{24}{3}$

(d) None of these

$$\int_0^2 x^2 + 3 = \left[\frac{x^3}{3} + 3x\right]_0^2 = \frac{8}{3} + 6 = \frac{26}{3}$$

$f'(x) = f(x)$

Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g be the function satisfying $f(x) + g(x) = x^2$.

The value of the integral $\int_0^1 f(x)g(x)\,dx$ is

$f(x) = ?$

$g(x) = ?$

$g(x) = x^2 - f(x)$

$g(x) = x^2 - e^x$

$$\int \frac{f'(x)}{f(x)} = \int 1$$

(A) $e - \frac{1}{2}e^2 - \frac{5}{2}$ (B) $e - e^2 - 3$ (C) $\frac{1}{2}(e-3)$ (D) $e - \frac{1}{2}e^2 - \frac{3}{2}$

$\log f(x) = x + c$

$f(x) = e^{x+c}$

$f(x) = e^{x+c}$

$f(0) = 1$

$1 = e^c$

$\boxed{c = 0}$

$f(x) = e^x$

$$\int_0^1 e^x(x^2 - e^x)\,dx = \int_0^1 x^2 e^x - \int_0^1 e^{2x}$$

$e^0 = 1$

$e^0 = 1$

$$\int_0^1 x^2 \cdot e^x = [x^2 \cdot e^x - 2x \cdot e^x + 2 \cdot e^x]_0^1 = e - \cancel{2e} + \cancel{2e} - 2$$

$$= e - 2$$

$e^0 = 1$

$\int_0^1 x^2 \cdot e^x = [x^2 \cdot e^x - 2x \cdot e^x + 2 \cdot e^x]_0 = e - 2e + 2e - 2$
$= e - 2$

$\int_0^1 e^{2x} = \left[\frac{e^{2x}}{2}\right]_0^1 = \frac{e^2}{2} - \frac{1}{2}$

$e - 2 - \frac{e^2}{2} + \frac{1}{2} = e - \frac{e^2}{2} - \frac{3}{2}$

$\int_0^{\frac{\pi}{2}} \frac{1+\cos 2x}{2}$

12. $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ **13.** $\int_2^3 \frac{x \, dx}{x^2+1}$ **14.** $\int_0^1 \frac{2x+3}{5x^2+1} dx$ **15.** $\int_0^1 x e^{x^2} dx$

16. $\int_1^2 \frac{5x^2}{x^2+4x+3}$ **17.** $\int_0^{\frac{\pi}{4}} (2\sec^2 x + x^3 + 2) \, dx$ **18.** $\int_0^{\pi} (\sin^2 \frac{x}{2} - \cos^2 \frac{x}{2}) \, dx$

$-\int_0^{\pi} \cos x$

19. $\int_0^2 \frac{6x+3}{x^2+4} dx$ **20.** $\int_0^1 (x e^x + \sin \frac{\pi x}{4}) \, dx$

Some Properties of Definite Integrals

$F(b) - F(a)$ $= -ve$
$F(a) - F(b)$

$\int_a^b$ $\int_b^a$

$x = t$
$dx = dt$

$\int_a^b f(x) \, dx = \int_a^b f(t) \, dt$

P₀ : $\int_a^b f(x) dx = \int_a^b f(t) \, dt$

P₁ : $\int_a^b f(x) dx = -\int_b^a f(x) dx$. In particular, $\int_a^a f(x) dx = 0$

P₂ : $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

P₃ : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

P₄ : $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

(Note that P₄ is a particular case of P₃)

P₅ : $\int_0^{2a} f(x) dx = \int_0^a f(x) dx + \int_0^a f(2a-x) dx$

P₆ : $\int_0^{2a} f(x) dx = 2\int_0^a f(x) dx$, if $f(2a-x) = f(x)$ and

0 if $f(2a-x) = -f(x)$

P₇ : (i) $\int_{-a}^a f(x) dx = 2\int_0^a f(x) dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^a f(x) dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

$2\int_0^a f(x) \, d.$

$0 \quad a \quad 2a$

$\int_0^{2a} f(x) \, dx = \int_0^a f(x) + \underbrace{\int_a^{2a} f(x) \, dx}$

Put $t = 2a - x$, $dt = -dx$

$-\int_a^0 f(2a-t) dt = \int_0^a f(2a-t) \, dt = \int_0^a f(2a-x) dx$

$= \int_0^a f(x) + \int_0^a f(2a-x) \, dx$

1. $\int_0^{\frac{\pi}{2}} \cos^2 x \, dx$ **2.** $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ **3.** $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x \, dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

$\frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}$

4. $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x \, dx}{\sin^5 x + \cos^5 x}$ **5.** $\int_{-5}^5 |x+2| \, dx$ **6.** $\int_2^8 |x-5| dx$

$|f(x)|$

$I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$

$I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2} - x\right) dx = \int_0^{\frac{\pi}{2}} \sin^2 x \, dx$

$+ve \; -ve$

$2I = \int_0^{\frac{\pi}{2}} 1 \, dx = [x]_0^{\frac{\pi}{2}}$

$2I = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

$I = \int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{2}-x\right)dx = \int_0^{\frac{\pi}{2}} \sin^2 x\, dx$ $\boxed{I = \frac{\pi}{4}}$

$\int_{-5}^{5} |x+2|\, dx = \int_{-5}^{-2} -(x+2) + \int_{-2}^{5}(x+2)\, dx$ (+ve, −ve; −5, −2, 5)

$x \neq 0$

$\frac{d}{dx}(|x|) = \frac{x}{|x|}$ $\frac{d}{dx}(|x|) = \left(\sqrt{x^2}\right)' = \frac{1}{2\sqrt{x^2}} \cdot 2x = \frac{x}{\sqrt{x^2}} = \frac{x}{|x|} = (|x|)'$

7. $\int_0^1 x(1-x)^n dx$ 8. $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$ 9. $\int_0^2 x\sqrt{2-x}\, dx$

10. $\int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x)\, dx$ 11. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx$

$1-x$

$I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)$

$I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right) = \int_0^{\frac{\pi}{4}} \log\left(1 + \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan\frac{\pi}{4}\cdot\tan x}\right)$

$= \int_0^{\frac{\pi}{4}} \log\left(\frac{\cancel{1+\tan x} + \cancel{1-\tan x}}{1+\tan x}\right) = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right)$

$I = \int_0^{\frac{\pi}{4}} \log 2 - \log(1+\tan x)\, dx$

$2I = \int_0^{\frac{\pi}{4}} \log 2 = \log 2 \cdot \frac{\pi}{4}$ $\boxed{I = \frac{\pi}{8}\cdot\log 2}$

$(\sin x)^7$ odd $\boxed{=0}$

$\cos^4 x \cdot \sin x$

12. $\int_0^{\pi} \frac{x\, dx}{1+\sin x}$ 13. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$ 14. $\int_0^{2\pi} \cos^5 x\, dx$

15. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x \cos x}\, dx$ 16. $\int_0^{\pi} \log(1+\cos x)\, dx$ 17. $\int_0^{a} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\, dx$

18. $\int_0^4 |x-1|\, dx$

P_3 P_4

Evaluate: $\int_0^{\pi/4} \sqrt{1 - \sin 2x}\, dx$ $\sqrt{\cos^2 x + \sin^2 x - 2\sin x\cos x}$

(a) $\sqrt{2} - 1$

(b) $\sqrt{2} + 1$

(c) $\sqrt{2}$

(d) None of these

$\int_0^{\frac{\pi}{4}} \sqrt{(\cos x - \sin x)^2} = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\, dx$

$\sin(A+B)$

$\sin(A)\cos B + \cos A \sin B$

$\left[\sin x + \cos x\right]_0^{\frac{\pi}{4}} = \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1\right] = \boxed{\sqrt{2} - 1}$

Evaluate: $\int_0^{2\pi} \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

Evaluate: $\int_0^{2\pi} \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) dx$

(a) $-2\sqrt{2}$

(b) -2

(c) $\sqrt{2}$

(d) $2\sqrt{2}$

$\int_0^{2\pi} \frac{1}{\sqrt{2}}\cos\frac{x}{2} + \frac{1}{\sqrt{2}}\sin\frac{x}{2} = \frac{1}{\sqrt{2}}\int_0^{2\pi} \cos\frac{x}{2} + \sin\frac{x}{2}$

$= \frac{1}{\sqrt{2}}\left[\frac{\sin x/2}{1/2} - \frac{\cos x/2}{1/2}\right]_0^{2\pi}$

$= \frac{2}{\sqrt{2}}\left[0+1+1\right] = 2\sqrt{2}$

$\boxed{\frac{2\tan\theta}{1+\tan^2\theta} = \sin(2\theta)}$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

Evaluate: $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

(a) $\frac{\pi}{2} - \log 2$

(b) π

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{2} - \log 2$

$x = \tan\theta$

$\theta = \tan^{-1} x$

$\theta = \tan^{-1}(0) = 0$

$\theta = \tan^{-1}(1) = \frac{\pi}{4}$

$\int_0^{\pi/4} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)\sec^2\theta\, d\theta$

$\int_0^{\pi/4} 2\theta \cdot \sec^2\theta\, d\theta$

$\int uv = u$

$\left[2\theta\cdot\tan\theta + 2\log(\cos x)\right]_0^{\pi/4}$

$= 2\cdot\frac{\pi}{4}\cdot 1 + 2\log\left(\frac{1}{\sqrt{2}}\right) - 0 - 2\log(1) = \frac{\pi}{2} + 2\left(\log(1) - \frac{1}{2}\log 2\right)$

$= \frac{\pi}{2} - \log 2$

$\boxed{\int \frac{\sin x}{x} = F(x)}$

Suppose that F(x) is an antiderivative of $f(x) = \frac{\sin x}{x}$, $x > 0$ then $\int_1^3 \frac{\sin 2x}{x} dx$ can be expressed as

$\int_a^b f(x)\,dx = \int_a^b f(t)\,dt$

(A) $F(6) - F(2)$ (B) $\frac{1}{2}(F(6) - F(2))$ (C) $\frac{1}{2}(F(3) - F(1))$ (D) $2(F(6) - F(2))$

$2x = t$

$2dx = dt$

$dx = \frac{dt}{2}$

if $x = 1$, $t = 2$

if $x = 3$, $t = 6$

$x = \frac{t}{2}$

$\int_2^6 \frac{\sin t}{t/2}\cdot\frac{dt}{2}$

$\int_2^6 \frac{\sin t}{t}\, dt$

$= \int_2^6 \frac{\sin x}{x}\, dx$

$= \left[F(x)\right]_2^6 = F(6) - F(2)$

+ve or −ve

−ve $x = 3$ +ve

+ve −ve

$x - 3$

$\frac{1}{2}$ 3 $\frac{7}{2} = 3.5$

$\int_{1/2}^{3\frac{1}{2}} \left\{\frac{1}{2}(|x-3| + |1-x| - 4)\right\} dx$ equals:

(A) $-\frac{3}{2}$ (B) $\frac{9}{8}$ (C) $-\frac{11}{4}$ (D) $\frac{3}{2}$

$\frac{1}{2}\left[\int_{1/2}^{3} -(x-3)\,dx + \int_3^{7/2}(x-3)\,dx + \int_1^{7/2}(1-x)\,dx + \int_{1/2}^{1} -(1-x) - \int_{1/2}^{7/2} 4\,dx\right]$

$\frac{25}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{25}{8}$ $+12$

$$\int_{-a}^{a} f(x)\,dx =$$

(A) $\int_0^a [f(x)+f(-x)]dx$ (B) $\int_0^a [f(x)-f(-x)]dx$ (C) $2\int_0^a f(x)\,dx$ (D) Zero

The value of the definite integral, $\int_0^{\sqrt{\ln\left(\frac{\pi}{2}\right)}} \cos\left(e^{x^2}\right)\cdot 2x\,e^{x^2}\,dx$ is

(A) 1 (B) $1+(\sin 1)$ (C) $1-(\sin 1)$ (D) $(\sin 1)-1$

Value of the definite integral $\int_{-1/2}^{1/2} \left(\sin^{-1}(3x-4x^3)-\cos^{-1}(4x^3-3x)\right)dx$

(A) 0 (B) $-\frac{\pi}{2}$ (C) $\frac{7\pi}{2}$ (D) $\frac{\pi}{2}$

Q10. $\int_0^{\pi} [\cot x]dx$, $[\bullet]$ denotes the greatest integer function, is equal to

A $\frac{\pi}{2}$

B 1

C -1

D $-\frac{\pi}{2}$

$I = \int_0^{\pi} [\cot(\pi - x)]dx$

$I = \int_0^{\pi} [-\cot x]dx$

$2I = \int_0^{\pi} [\cot x] + [-\cot x]\,dx$

$[x]+[-x] = -1$

$$\int_0^{\pi}(-1)\,dx = [-x]_0^{\pi} = -\pi$$

Q7. If for all real triplets (a,b,c), $f(x) = a+bx+cx^2$, then $\int_0^1 f(x)\,dx$ is equal to :

$$\int_0^1 a+bx+cx^2 = \left[ax+\frac{bx^2}{2}+\frac{cx^3}{3}\right]_0^1$$

$$\boxed{= a+\frac{b}{2}+\frac{c}{3}}$$

A $\frac{1}{3}\left\{f(0)+f\left(\frac{1}{2}\right)\right\}$

B $\frac{1}{3}\left\{f(1)+3f\left(\frac{1}{2}\right)\right\}$

C $\frac{1}{6}\left\{f(0)+f(1)+4f\left(\frac{1}{2}\right)\right\}$

D $2\left\{3f(1)+2f\left(\frac{1}{2}\right)\right\}$

$\frac{1}{6}\left(a+a+b+c+4\left(a+\frac{b}{2}+\frac{c}{4}\right)\right) = \boxed{a+\frac{b}{2}+\frac{c}{3}}$

$f(0) = a$

$f(1) = a+b+c$

$f\left(\frac{1}{2}\right) = a+\frac{b}{2}+\frac{c}{4}$

$2^{-x} = \left(\frac{1}{2^x}\right)$

Q2. The value of $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x}\,dx$ is :

$$I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2(-x)}{1+2^{-x}} = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+\frac{1}{2^x}}$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{2^x \sin^2 x}{2^x+1}$$

A $\frac{\pi}{4}$

B $\frac{\pi}{8}$

C $\frac{\pi}{2}$

D 4π

$$2I = \int_{-\pi/2}^{\pi/2} \frac{\sin^2 x(1+2^x)}{1+2^x} = \int_{-\pi/2}^{\pi/2} \sin^2 x$$

$$2I = 2\int_0^{\pi/2} \sin^2 x$$

$$I = \frac{1}{2}\int_0^{\pi/2} 1-\cos 2x\,dx = \frac{1}{2}\left[x-\frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{4}$$

$\sin \pi = 0$

Q3. The integration $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to:

$$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos(\pi-x)} = \int_{\pi/4}^{3\pi/4} \frac{dx}{1-\cos x}$$

A -2

A -2

B 2

C 4

D -1

$I = \int_{-\pi/4}^{3\pi/4} \cos^2 x \, dx$

$2I = \int_{-\pi/4}^{3\pi/4} \frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

$\frac{1-\cos x + 1+\cos x}{1-\cos^2 x} = \frac{2}{1-\cos^2 x}$

Q4. The integral $\int_2^4 \frac{\log x^2}{\log x^2+\log(36-12x+x^2)} dx$ is equal to :

$\log(6-x)^2 + \log(36-12(6-x)+(6-x)^2)$

A 4

B 1

C 6

D 2

$I = \int_2^4 \frac{\log(x^2-12x+36)}{\log(x^2-12x+36)+\log(x^2)}$

$2I = \int_2^4 \frac{\log x^2 + \log(x^2-12x+36)}{\log x^2 + \log(x^2-12x+36)}$

$2I = \int_2^4 1 \, dx = [x]_2^4$

$2I = 2$

$\boxed{I = 1}$

Q7. The value of the integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}} dx$ is

A 1/2

B 3/2

C 2

D 1

Q10. The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$

$\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1+2^x}$

A π

B $a\pi$

C $\frac{\pi}{2}$

D 2π

D 2π

The value of the definite integral, $\int_1^{\infty}(e^{x+1}+e^{3-x})^{-1}\,dx$ is

(A) $\frac{\pi}{4e^2}$ (B) $\frac{\pi}{4e}$ (C) $\frac{1}{e^2}\left(\frac{\pi}{2}-\tan^{-1}\frac{1}{e}\right)$ (D) $\frac{\pi}{2e^2}$

$$\int_1^{\infty}(e\cdot e^{x}+e^{3}\cdot e^{-x})^{-1}\,dx$$

$$\int_1^{\infty}\left(e\cdot e^{x}+\frac{e^3}{e^x}\right)^{-1} = \int_1^{\infty}(e\cdot e^{2x}+e^{3})^{-1}\cdot e^{x}\,dx$$

$$= \frac{1}{e}\int_1^{\infty}(e^{2x}+e^{2})^{-1}\cdot e^{x}\,dx = \frac{1}{e}\int\frac{e^{x}\,dx}{e^{2x}+e^{2}}$$

$e^{x}=t$, $e^{x}\,dx=dt$

$$\frac{1}{a}\tan^{-1}\frac{t}{a} \qquad \frac{1}{t^2+a^2}$$

$$= \frac{1}{e}\int_e^{\infty}\frac{dt}{t^2+e^2}$$

$$= \frac{1}{e}\cdot\frac{1}{e}\left[\tan^{-1}\left(\frac{t}{e}\right)\right]_e^{\infty}$$

$$= \frac{1}{e^2}\left[\frac{\pi}{2}-\frac{\pi}{4}\right] = \frac{\pi}{4e^2}$$

If $f(x)=e^{g(x)}$ and $g(x)=\int_2^x\frac{t\,dt}{1+t^4}$ then $f'(2)$ has the value equal to :

(A) 2/17 (B) 0 (C) 1 (D) cannot be determined

$$\frac{d}{dx}\left(\int_2^x\frac{t\,dt}{1+t^4}\right) = \frac{x}{1+x^4}\cdot 1 - \frac{2}{1+2^4}\cdot 0 = \frac{x}{1+x^4}$$

① $\frac{d}{dx}\left(\int f(x)\right) = f(x)$

② $\frac{d}{dx}\left(\int_{h(x)}^{g(x)} f(t)\,dt\right) = f(g(x))\cdot\frac{d}{dx}g(x) - f(h(x))\cdot\frac{d}{dx}(h(x))$

$$= \frac{x}{1+x^4}\cdot 1 - \frac{2}{1+2^4}\cdot 0$$

$\int_a^a = 0$

$$f'(2) = e^{\int_2^2\frac{t\,dt}{1+t^4}}\cdot\frac{2}{1+2^4} = e^{0}\cdot\frac{2}{17} = \frac{2}{17}$$

If x satisfies the equation $\left(\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right)x^2 - \left(\int_{-3}^{3} \frac{t^2 \sin 2t}{t^2+1}dt\right)x - 2 = 0$ $(0 < \alpha < \pi)$, then the value x is

(A) $\pm\sqrt{\frac{\alpha}{2\sin\alpha}}$ (B) $\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$ (C) $\pm\sqrt{\frac{\alpha}{\sin\alpha}}$ (D) $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

$\int \cos^m x \, dx$

odd = 0

Let $I_1 = \int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x.\cos x}dx$; $I_2 = \int_0^{2\pi}(\cos^6 x)dx$; $I_3 = \int_{-\pi/2}^{\pi/2}(\sin^3 x)dx$ & $I_4 = \int_0^1 ln\left(\frac{1}{x}-1\right)dx$ then

(A) $I_1 = I_2 = I_3 = I_4 = 0$ (B) $I_1 = I_2 = I_3 = 0$ but $I_4 \neq 0$
(C) $I_1 = I_3 = I_4 = 0$ but $I_2 \neq 0$ (D) $I_1 = I_2 = I_4 = 0$ but $I_3 \neq 0$

$\int \frac{1-x^7}{x(1+x^7)}$ dx equals :

(A) $ln\, x + \frac{2}{7}\, ln\,(1+x^7) + c$ (B) $ln\, x - \frac{2}{7}\, ln\,(1-x^7) + c$

(C) $ln\, x - \frac{2}{7}\, ln\,(1+x^7) + c$ (D) $ln\, x + \frac{2}{7}\, ln\,(1-x^7) + c$

The value of the integral $\int_{-\pi}^{\pi}(\cos px - \sin qx)^2\, dx$ where p, q are integers, is equal to :

(A) $-\pi$ (B) 0 (C) π (D) 2π